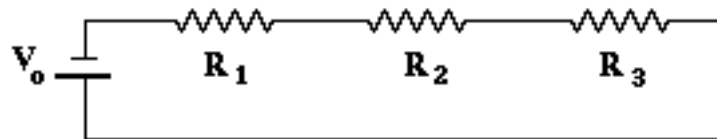


Calculating Resistance in a Circuit

We can use Kirchhoff's Laws in combination with Ohm's Law to find the total resistance in circuits.

Resistors in Series

Consider the diagram below.



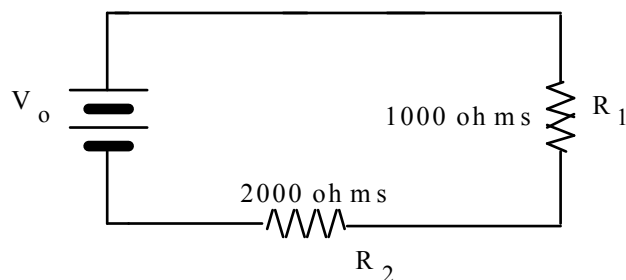
The voltage law states that $V_o = V_1 + V_2 + V_3$, and the current law states that the current is the same through each resistor. Substituting Ohm's Law into this equation gives:

$$IR_o = IR_1 + IR_2 + IR_3$$

Since the current cancels out on both sides we have the equation for finding the total resistance in series:

$$R_o = R_1 + R_2 + R_3 + \dots \text{etc.}$$

Example #7: Consider the following circuit diagram showing two resistors attached in series to a battery of two 1.5 V cells. Determine all unknown voltages, currents and resistances for each apparatus in the circuit.



$V_o = \text{p.d. of 2-1.5 cells in series}$

Since cells connected in series have their voltages added together, the total voltage,

$$\begin{aligned} V_o &= 2 \times 1.5 \text{ V} \\ &= 3.0 \text{ V} \end{aligned}$$

Some helpful hints:

- First find the total effect of resistance, R_o . This should be listed with the power supply.
- Second, use Ohm's Law to determine the current I_o supplied by the battery.
- Next, use Kirchhoff's Law to list the current through each resistor.
- Finally, use Ohm's Law again to find the voltage drop across each resistor; note that the sum of these voltages = V_o .

(see Circuitry Ex 7 for answer)

In summary, for a *series* circuit,

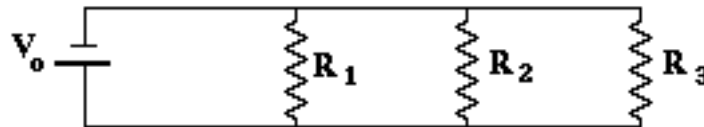
$$V_o = V_1 + V_2 + \dots \text{etc. (voltage is added)}$$

$$I_o = I_1 = I_2 = \dots \text{etc. (current is the same throughout the circuit)}$$

$$R_o = R_1 + R_2 + \dots \text{etc. (resistor values are added)}$$

Resistors in Parallel

Consider this new circuit diagram, showing three resistors in parallel with a power supply of voltage V_o .



The current law states that current generated at V_o will split so that some of the total current I_o goes through each resistor. In effect,

$$I_o = I_1 + I_2 + I_3.$$

The voltage law states that the voltage drop is the same through each resistor. Substituting Ohm's Law into this equation gives:

$$\frac{V}{R_o} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

Since the voltage cancels out on both sides we have the equation for finding the total resistance in series:

$$\frac{1}{R_o} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

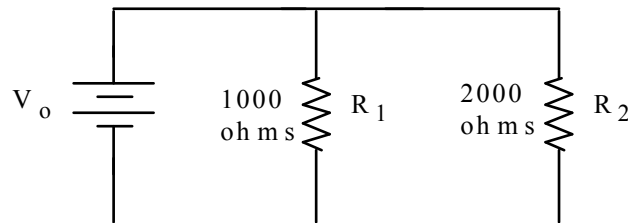
Example #8: Use your calculator to add these resistors in parallel:

(a) 25 Ω , 30 Ω , 50 Ω

(b) 50 Ω , 68 Ω , 270 Ω , 569 Ω

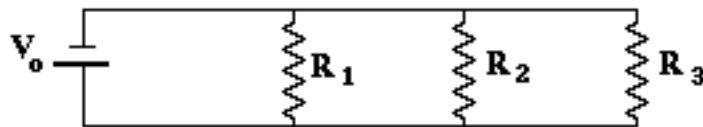
(see Circuitry Ex 8 for answer)

Example #9: Consider the following circuit diagram showing two resistors attached in parallel to a battery of two 1.5 V cells. Determine all unknown currents, voltages and resistances.



(see Circuitry Ex 9 for answer)

Example #10: In this example $R_1 = 5.0 \, \Omega$, $R_2 = 10 \, \Omega$ and $R_3 = 15 \, \Omega$ and the total current is 10 A. Find the current in each branch.



(see Circuitry Ex 10 for answer)

In summary, for a *parallel* circuit,

$$V_o = V_1 = V_2 = \dots \text{etc.}$$

$$I_o = I_1 + I_2 + \dots \text{etc.}$$

$$\frac{1}{R_o} = \frac{1}{R_1} + \frac{1}{R_2} + \dots \text{etc.}$$

Also note the following:

- When resistors are added in series the total resistance is always *larger* than the largest resistor.
- When resistors are added in parallel the total resistance is always *smaller* than the smallest resistor.

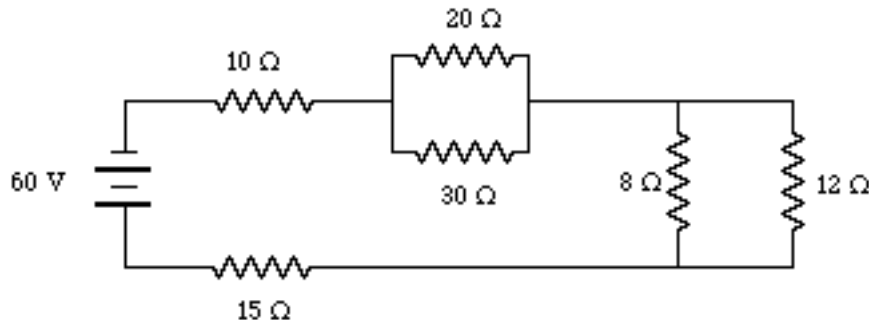
Finally, you have probably noticed that most circuits contain a mixture of resistors in series and in parallel. Such arrangements are called *combination* circuits.

In combination circuits, we want to find:

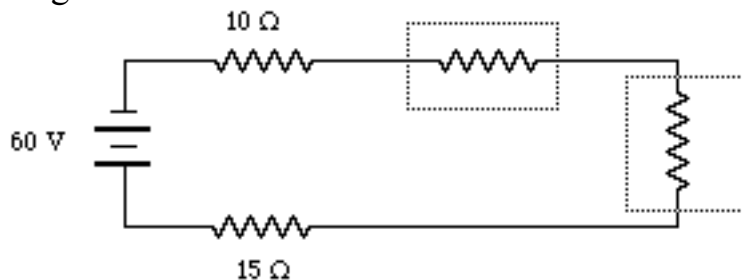
- the equivalent resistance for various parts of the circuit;
- the total resistance **R_o** in the circuit;
- the current flow through each device in the circuit;
- the voltage gain or drop through each device in the circuit.

In performing these tasks, you will use a combination of Ohm's Law and Kirchhoff's Laws. Ohm's Law will apply for an entire circuit if the total values are used, or apply to an individual resistor if the individual values are used.

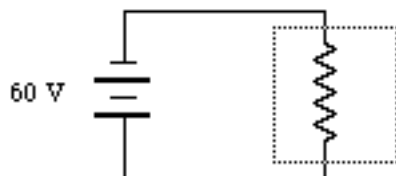
The following are the steps to use to solve a combination circuit:



- Reduce the individual resistor networks to the equivalent of one resistor by applying the appropriate equations. You should now have only a group of series resistors. Redraw the circuit in this simplified way. Determine the missing resistances below:



- Add these series resistors to get the equivalent of one resistor. Determine this value:



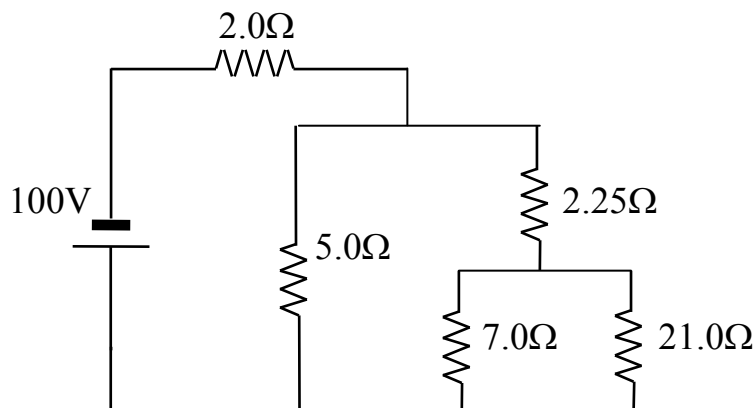
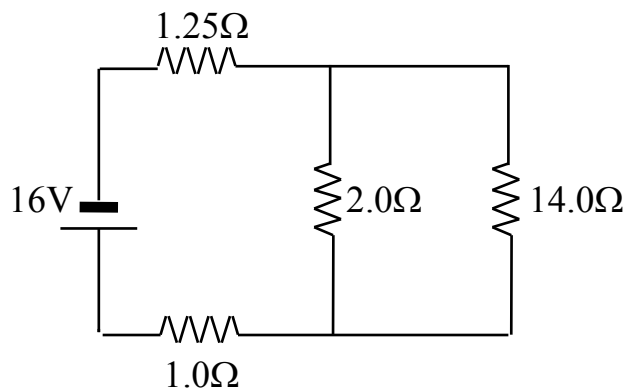
- Find the current in the circuit using Ohm's Law.
- Finally, work backwards through the circuit using Ohm's Law and/or Kirchhoff's Laws to find the current and voltage across each resistor.

Example #11: Complete the table for the circuit on the previous page:

R (Ω)	V (V)	I (A)
10	_____	_____
20	_____	_____
30	_____	_____
8	_____	_____
12	_____	_____
15	_____	_____

(see Circuitry Ex 11 for answer)

Example #12: Calculate all unknown voltages, currents and resistances for the following circuits:



(see Circuitry Ex 12 for answer)